Published by Institute of Physics Publishing for SISSA

Received: *June 14, 2006*
Accepted: *July 17, 2006*
Published: *August 16, 2006*

Space-like branes, accelerating cosmologies and the near 'horizon' limit

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Abstract: It is known that there exist two different classes of time dependent solutions in the form of space-like (or S)-branes in the low energy M/string theory. Accelerating cosmologies are known to arise from S-branes in one class, but not in the other where the time-like holography in the dS/CFT type correspondence may be more transparent. We show how the accelerating cosmologies arise from S-branes in the other class. Although we do not get the de Sitter structure in the lowest order supergravity, the near 'horizon' ($t \to 0$) limits of these S-branes are the generalized Kasner metric.

Keywords: M-Theory, p-branes, Superstrings and Heterotic Strings.

The interests in the time-dependent solutions in low energy (dimensionally reduced) string/M theory are manifold. (a) They might lead to a better understanding of the black hole as well as big bang singularities. (b) They might tell us about how the cosmological observations of our universe can be understood from a fundamental theory. (c) The time dependent solutions can provide a concrete realization of dS/CFT correspondence and show us how time emerges from an Euclidean world-brane theory. (d) They can help us understand the time dependent processes in string theory.

Low energy string/M theory admits various kinds of time dependent solutions. We here consider a specific kind of solutions which are non-supersymmetric, singular and has the form of space-like (or S)-branes [1][1]. An Sp-brane has a $(p+1)$-dimensional Euclidean world-volume and has isometry $\mathrm{ISO}(p+1) \times \mathrm{SO}(d-p-2,1)$ in d space-time dimensions. In the literature there exist two classes of S-brane solutions. In one class the solutions are asymptotically ($t \to \infty$) non-flat and are characterized by three or more independent parameters [3, 4]. The coordinates used here are also quite different from the usual BPS p-branes. Whereas, in the other class the solutions are asymptotically ($t \to \infty$) flat and are characterized by two or more parameters [5][2]. Here the solutions can be written in terms of a single harmonic function with a singularity at $t = 0$ much like a BPS p-brane (of course the form of the solutions are quite different) with the radial coordinate r taking the place of time t. As the AdS/CFT correspondence is well-understood for BPS D3-brane, it is hoped that for this class of solutions, the dS/CFT correspondence [7−9][3] can similarly be understood by taking a near 'horizon'[4] limit. However, since it is difficult to find de Sitter solution in low energy string theory[5], the understanding of this issue remains unclear. The possibilities of obtaining eternally accelerating solutions from S-branes were also discussed in [12].

One of our objectives here is to find the four dimensional cosmologies from the S-brane solutions just mentioned. For this purpose let us recall that for the static BPS p-brane solutions combining the radial part with the brane world-volume part we get a $(p+2)$ dimensional geometry which can be thought of as obtained by compactifying the ten dimensional theory on a $(8-p)$-dimensional sphere. In the similar spirit, for S-branes, combining the temporal part with the $(p+1)$-dimensional Euclidean brane world-volume part, we get a $(p+2)$-dimensional geometry by compactifyting the d dimensional theory on $(d-p-2)$ dimensional hyperbolic space. So, in order to get a four dimensional world, we must take $p = 2$. In general the volume of compactification associated with the hyperbolic space is time dependent and for this case it is possible to obtain accelerating cosmologies in four dimensions, evading a 'no-go' theorem [13, 14], as was shown

[1]See [2] for some earlier works on time dependent solutions in supergravities.

[2]In ten dimensions this class of solutions was also obtained in [6].

[3]See [10] for some earlier works on dS/CFT correspondence.

[4]This is really an abuse of the term "horizon", but we are using it in analogy with the static, BPS p-brane solutions of M/string theory. Even for the latter case, as is well-known, it is degenerate with zero area.

[5]Some such solutions were found in [11] with the inclusion of higher order curvature terms in the effective action.

by Townsend and Wohlfarth [15] starting from pure Einstein gravity in higher dimensions. The solution used in [15] is a special case of the first class of S-brane solutions we mentioned above, that is, they are asymptotically non-flat. In general in this case one should have a two parameter family of solutions [3], but ref. [15] used some specific values of the parameters. A more general S-brane solutions containing a gauge field (and a dilaton) and belonging again to the first class was used in refs. [16, 17] to obtain four dimensional cosmology[6]. These solutions in general contains three (four) independent parameters and in showing the accelerating cosmology the parameters are restricted to specific values. When the gauge field (and the dilaton) is (are) put to zero these solutions reduce to those of Townsend and Wohlfarth when the parameters take specific values. Accelerating cosmologies were not known to follow from the second class of S-brane solutions[7] where the coordinates are much like those of BPS p-branes and dS/CFT type correspondence may be easily understood. So in this paper we will show that four dimensional accelerating cosmologies also follow from the second class of S-brane solutions where the solutions are asymptotically flat and are characterized by two or more independent parameters.

The second class of d-dimensional Sp-brane solutions were constructed in [5]. They can also be obtained by a Wick rotation of the static, non-susy p-brane solutions obtained in [20] as was shown there. Substituting $p = 2$ and considering the $d - 4 = n$-dimensional hyperbolic space compactification [21 − 23] on time varying volume we obtain the four dimensional metric in the Einstein frame. We show that the metric represents flat, homogeneous and isotropic FLRW universe with some scale factor $S(\eta)$. Then we show that for certain values of the parameters, we can get an accelerating expansion from the four dimensional metric which can be thought of as obtained from time dependent hyperbolic compactifications of low energy M or string theory for $n = 7$ or 6. It thus shows that the accelerating cosmologies quite generically follow from the S-branes irrespective of the fact whether they are asymptotically flat or not. Here also as for the other class of S-branes, the acceleration is transient with two decelerating phases at $\eta \to 0$ and $\eta \to \infty$ and does not lead to a realistic cosmology. We find that as $\eta \to 0$, the behavior of the scale factor is universal with $S(\eta) \sim \eta^{1/3}$ irrespective of whether we consider first class or second class of S-brane solutions and whether we have a gauge field and/or a dilaton or not. The reason for this can be attributed to the fact that the near 'horizon' or $\eta \to 0$ limit of these solutions give the generalized Kasner metric[8].

The asymptotically flat, space-like or S2-brane solution in gravity coupled to dilaton and an $(n - 1)$-form gauge field in $n + 4$ space-time dimensions is given in [5] and has the

[6]Cosmological space-times from S-branes were also studied earlier in ref. [18].

[7]In fact it might seem that the accelerating cosmologies do not follow from this class of S-brane solutions. The reason is, although it is known [19] that the first class of solutions maps to the second class under a coordinate transformation on imposing the same boundary conditions on the metric and the dilaton but, for the parameter restrictions used in refs. [15 − 17] these two classes remains disctinct. So, it may appear that the asymptotic non-flatness of the metric may be a necessary condition for the appearance of accelerating cosmologies. We will show later that this is not correct.

[8]This was also noticed in a different context in the second paper of reference [4] and [24]. We would like to thank Vladimir D. Ivashchuk for pointing this out to us.

form,

$$
\begin{aligned}
ds^2 &= F^{\frac{12}{(n-1)\chi}} \left(H\tilde{H}\right)^{\frac{2}{n-1}} \left(-dt^2 + t^2 dH_n^2\right) + F^{-\frac{4}{\chi}} \sum_{i=1}^{3} (dx^i)^2 \\
e^{2\phi} &= F^{-\frac{4a(n+2)}{(n-1)\chi}} \left(\frac{H}{\tilde{H}}\right)^{2\delta} \\
F_{[n]} &= b\mathrm{Vol}(H_n)
\end{aligned} \tag{1}
$$

In the above $F = \left(H(t)/\tilde{H}(t)\right)^{\alpha} \cos^2\theta + \left(\tilde{H}(t)/H(t)\right)^{\beta} \sin^2\theta$, where, $H(t) = 1 + \omega^{n-1}/t^{n-1}$ and $\tilde{H}(t) = 1 - \omega^{n-1}/t^{n-1}$ are the two harmonic functions. The physically acceptable region is $t > \omega$. Here α, β, θ, ω and δ are integration constants. b is a 'charge' parameter and dH_n^2 represents the line element of an n-dimensional hyperbolic space and $\mathrm{Vol}(H_n)$ is its volume form. a is the dilaton coupling which is given by $a^2 = 4 - 6(n-1)/(n+2)$ for maximal supergravities [25] in diverse dimensions $d = n + 4$. Note that $a = 0$ for $n = 7$ or for M-theory and $a = -1/2$ for space-like D2-brane solutions of string theory. Also in the above χ is defined to be $\chi = 6 + a^2(n+2)/(n-1)$. Note that as $t \to \infty$, the functions $H(t)$, $\tilde{H}(t)$ and F go to unity and so the metric becomes asymptotically flat in Rindler coordinates. We mentioned that the solution depend on several parameters but not all of them are independent. They are related as

$$
\begin{aligned}
\alpha - \beta &= a\delta \\
\frac{1}{2}\delta^2 + \frac{2\alpha(\alpha - a\delta)(n+2)}{\chi(n-1)} &= \frac{n}{n-1} \\
b &= \sqrt{\frac{4(n+2)(n-1)}{\chi}} (\alpha + \beta)\omega^{n-1} \sin 2\theta
\end{aligned} \tag{2}
$$

So, the only independent paramaters are ω, θ and δ (δ vanishes for $n = 7$).

Now instead of writing the solution in terms of two harmonic functions we can rewrite it in terms of a single harmonic function much like the BPS p-branes. For this purpose we make a coordinate transformation,

$$
\tilde{t} = t\left(1 + \frac{\omega^{n-1}}{t^{n-1}}\right)^{\frac{2}{n-1}} = tH^{\frac{2}{n-1}} \tag{3}
$$

This relation can be inverted to give

$$
t = \tilde{t}\left(\frac{1 + \sqrt{f}}{2}\right)^{\frac{2}{n-1}} \tag{4}
$$

where $f(\tilde{t}) = 1 - 4\omega^{n-1}/\tilde{t}^{n-1}$. The function $F(t)$ above can then be written as

$$
F = \left(\frac{H}{\tilde{H}}\right)^{\alpha} \cos^2\theta + \left(\frac{\tilde{H}}{H}\right)^{\beta} \sin^2\theta = f^{-\frac{\alpha}{2}} \cos^2\theta + f^{\frac{\beta}{2}} \sin^2\theta \tag{5}
$$

The solution (1) can then be rewritten as,

$$
\begin{aligned}
ds^2 &= F^{\frac{12}{(n-1)\chi}} f^{\frac{1}{n-1}} \left(-\frac{d\tilde{t}^2}{f} + \tilde{t}^2 dH_n^2\right) + F^{-\frac{4}{\chi}} \sum_{i=1}^{3} (dx^i)^2 \\
e^{2\phi} &= F^{-\frac{4a(n+2)}{(n-1)\chi}} f^{-\delta} \\
F_{[n]} &= b\mathrm{Vol}(H_n)
\end{aligned}
\tag{6}
$$

By defining another coordinate

$$
\hat{t}^{n-1} = \tilde{t}^{n-1} - 4\omega^{n-1} \tag{7}
$$

we further rewrite the solution as

$$
\begin{aligned}
ds^2 &= \left(g^{\frac{\alpha}{2}} \cos^2\theta + g^{-\frac{\beta}{2}} \sin^2\theta\right)^{\frac{12}{(n-1)\chi}} g^{\frac{1}{n-1}} \left(-\frac{d\hat{t}^2}{g} + \hat{t}^2 dH_n^2\right) \\
&\qquad + \left(g^{\frac{\alpha}{2}} \cos^2\theta + g^{-\frac{\beta}{2}} \sin^2\theta\right)^{-\frac{4}{\chi}} \sum_{i=1}^{3} (dx^i)^2 \\
e^{2\phi} &= \left(g^{\frac{\alpha}{2}} \cos^2\theta + g^{-\frac{\beta}{2}} \sin^2\theta\right)^{-\frac{4a(n+2)}{(n-1)\chi}} g^{\delta} \\
F_{[n]} &= b\mathrm{Vol}(H_n)
\end{aligned}
\tag{8}
$$

where $g(\hat{t}) = 1 + 4\omega^{n-1}/\hat{t}^{n-1}$. This solution has a singularity at $\hat{t} = 0$ much like the static, BPS p-brane which has singularity at $r = 0$, otherwise it is regular everywhere. The various parameters like α, β, ω, θ, δ and b satisfy the same relations as given before in eq. (2). Also in the following we will replace $\hat{t}$ in solution (8) by t for brevity. As mentioned before in order to obtain four dimensional cosmology we combine the temporal part with the three dimensional Euclidean world-volume part and for obtaining the four dimensional cosmology in Einstein frame we extract from the four dimensional part the proper conformal factor. So, we write the metric in (8) as follows,

$$
ds^2 = F^{-\frac{6n}{(n-1)\chi}} g^{-\frac{n}{2(n-1)}} t^{-n} ds_E^2 + F^{\frac{12}{(n-1)\chi}} g^{\frac{1}{n-1}} t^2 dH_n^2 \tag{9}
$$

where

$$
ds_E^2 = -F^{\frac{6(n+2)}{(n-1)\chi}} g^{-\frac{n-4}{2(n-1)}} t^n dt^2 + F^{\frac{2(n+2)}{(n-1)\chi}} g^{\frac{n}{2(n-1)}} t^n \sum_{i=1}^{3} (dx^i)^2 \tag{10}
$$

is the four dimensional metric in the Einstein frame. Here $F(t) = g(t)^{\alpha/2} \cos^2\theta + g(t)^{-\beta/2} \times \sin^2\theta$, with $g(t)$ as given before. Now redefining a new time coordinate η by

$$
d\eta = F^{\frac{3(n+2)}{(n-1)\chi}} g^{-\frac{n-4}{4(n-1)}} t^{\frac{n}{2}} dt \tag{11}
$$

we can write the Einstein frame metric ds_E^2 in the standard FLRW form as

$$
ds_E^2 = -d\eta^2 + S^2(\eta) \left((dx^1)^2 + (dx^2)^2 + (dx^3)^2\right) \tag{12}
$$

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where the scale factor $S(\eta)$ has the form,

$$S(\eta) \equiv A(t) = F^{\frac{n+2}{(n-1)\chi}} g^{\frac{n}{4(n-1)}} t^{\frac{n}{2}} \tag{13}$$

Let us also define another function

$$B(t) = F^{-\frac{2(n+2)}{(n-1)\chi}} g^{\frac{n-2}{2(n-1)}} \tag{14}$$

We will have an expanding universe if the scale factor satisfies $dS(\eta)/d\eta > 0$ and the expansion will be accelerated if $d^2S(\eta)/d\eta^2 > 0$. In terms of the functions $A(t)$ and $B(t)$, these two conditions can be written as

$$\begin{aligned} m(t) &\equiv \frac{d \ln A}{dt} \;>\; 0 \\ n(t) &\equiv \frac{d^2 \ln A}{dt^2} + \frac{d \ln A}{dt}\frac{d \ln B}{dt} \;>\; 0 \end{aligned} \tag{15}$$

Now in order to understand whether (12) can give an accelerating cosmology or not we will have to fix the various parameters and study the functions $m(t)$ and $n(t)$ of eqs. (15) to see whether both can be satisfied for some range of the time coordinate t or η. We first note that from the second relation in eq. (2) we get α and β in terms of δ as

$$\begin{aligned} \alpha &= \pm\sqrt{\frac{\chi n - 3\delta^2(n-1)}{2(n+2)}} + \frac{a\delta}{2} \\ \beta &= \pm\sqrt{\frac{\chi n - 3\delta^2(n-1)}{2(n+2)}} - \frac{a\delta}{2} \end{aligned} \tag{16}$$

In the following we will study the four cases separately.

(*a*) Pure Einstein gravity ($a = 0$, $\delta = 0 \Rightarrow \alpha = \beta$, $b = 0 \Rightarrow \theta = 0$)

(*b*) Einstein gravity with an $(n-1)$-form gauge field ($\delta = 0 \Rightarrow \alpha = \beta$, $a = 0$, $b \neq 0$)

(c) Einstein gravity with a dilaton ($a \neq 0$, $b = 0 \Rightarrow \theta = 0$)

(*d*) Einstein gravity with a dilaton and an $(n-1)$-form gauge field.

We will take $n = 7$ for M-theory compactification (Case I) and $n = 6$ for string theory compactification (Case II). We will discuss the above four possibilities in these two cases. Note that for $n = 7$, there is no dilaton and so, $a = \delta = 0$ and therefore cases (c) and (d) do not arise. But for string theory all these four cases will arise.

Case I: M-theory compactifications ($n = 7$)

(a) For this case $a = 0$, $\delta = 0$ and $\chi = 6$ and so, $\alpha = \beta = \pm\sqrt{7/3}$. Also, we have $b = \theta = 0$, so there is no gauge field. The function $F = g(t)^{\alpha/2}$, so, from (13) and (14) we find

$$\begin{aligned} A &= g(t)^{\pm\frac{1}{8}\sqrt{\frac{7}{3}}+\frac{7}{24}} t^{\frac{7}{2}} \\ B &= g(t)^{\mp\frac{1}{4}\sqrt{\frac{7}{3}}+\frac{5}{12}} \end{aligned} \tag{17}$$

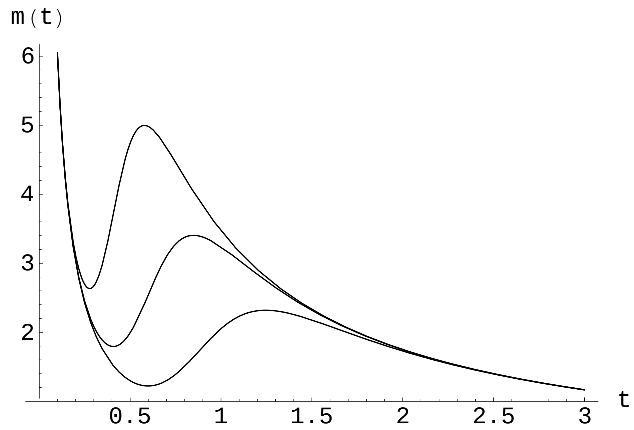

Figure 1: Plot of $m(t)$ for M-theory compactification without a gauge field ($\theta = 0$) given in (15) for $4\omega^6 = 0.01$, 0.1, and 1. The top curve is for the value 0.01, the middle one for 0.1 and so on. All the curves meet at small and large values of t.

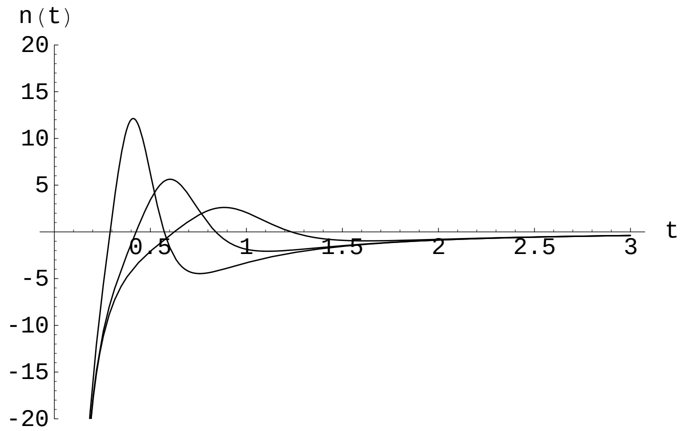

Figure 2: Plot of $n(t)$ for M-theory compactification without a gauge field ($\theta = 0$) given in (15) for $4\omega^6 = 0.01$, 0.1, and 1. The left curve is for the value 0.01, the middle one for 0.1 and so on. All the curves appear to meet at late and early times. All the curves are positive for some finite interval of time indicating an acceleration for that period, but are decelerating beyond that.

where $g(t) = 1 + 4\omega^6/t^6$. We have plotted the functions $m(t)$ and $n(t)$ defined in (15) in figures 1,2 for the upper sign and have shown that both the conditions in (15) can be simultaneously satisfied for certain range of t indicating an accelerating cosmology for that period of time. The lower sign does not produce accelerated expansion. Note that we have plotted $m(t)$ and $n(t)$ for $4\omega^6 = 0.01$, 0.1, and 1 to show the behavior of the functions for different values of the parameters. The parameter $4\omega^6$ has very little effect at very large and very low values of t. This is the reason we see acceleration only for a finite interval of time. When we introduce more and more parameters to study various other M/string theory compactifications we will see that the basic behavior of the functions remain the same and the additional parameters do not contribute significantly to the cosmology we obtain.

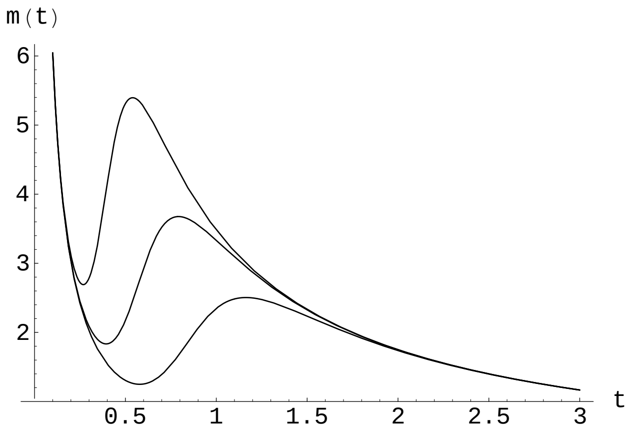

Figure 3: Plot of $m(t)$ given in (15) for M-theory compactification with a gauge field for $4\omega^6 = 0.01$, 0.1, and 1 and $\theta = \pi/4$. The top curve is for the value 0.01, the middle one for 0.1 and so on. All the curves meet at small and large values of t.

(b) For this case $a = \delta = 0$ and so, $\alpha = \beta = \pm\sqrt{7/3}$ as in the previous case. However, here $b \neq 0$ which implies $\theta \neq 0$, that is, there is a non-zero 6-form gauge field. Here also $\chi = 6$. We have considered three different values of θ i.e. $\theta = \pi/4$, $\pi/6$, and $\pi/18$. However we find that the functions behave almost similarly for different values of θ and so, we give here the plots only for $\theta = \pi/4$. In this case the various functions have the forms,

$$\begin{aligned} F(t) &= \frac{1}{2}\left(g(t)^{\frac{1}{2}\sqrt{\frac{7}{3}}} + g(t)^{-\frac{1}{2}\sqrt{\frac{7}{3}}}\right) \\ A(t) &= \frac{1}{2^{\frac{1}{4}}}\left(g(t)^{\frac{1}{2}\sqrt{\frac{7}{3}}} + g(t)^{-\frac{1}{2}\sqrt{\frac{7}{3}}}\right)^{\frac{1}{4}} g(t)^{\frac{7}{24}}\, t^{\frac{7}{2}} \\ B(t) &= 2^{\frac{1}{2}}\left(g(t)^{\frac{1}{2}\sqrt{\frac{7}{3}}} + g(t)^{-\frac{1}{2}\sqrt{\frac{7}{3}}}\right)^{-\frac{1}{2}} g(t)^{\frac{5}{12}} \end{aligned} \tag{18}$$

where $g(t)$ is as given in (a) above. Here also we have plotted the functions $m(t)$ and $n(t)$ given in (15) for $4\omega^6 = 0.01$, 0.1, 1 in figures 3,4. Again the plots show that we get an accelerating cosmology for certain interval of time.

Case II: String theory compactifications ($n = 6$)[9]

(a) For this case $a = \delta = 0$, which implies $\alpha = \beta = \pm 3/2$. Also $\chi = 6$ and $b = \theta = 0$. The various functions in this case have the forms

$$\begin{aligned} F(t) &= g(t)^{\pm\frac{3}{4}} \\ A(t) &= g(t)^{\pm\frac{1}{5}+\frac{3}{10}}\, t^3 \\ B(t) &= g(t)^{\mp\frac{2}{5}+\frac{2}{5}} \end{aligned} \tag{19}$$

where $g(t) = 1 + 4\omega^5/t^5$. We have plotted $m(t)$ and $n(t)$ using the functions in (19). But since we have pure Einstein gravity we expect the functions to have the same behavior as

[9] Unlike in cases (c), (d) cases (a), (b) given below do not correspond to maximal supergravities.

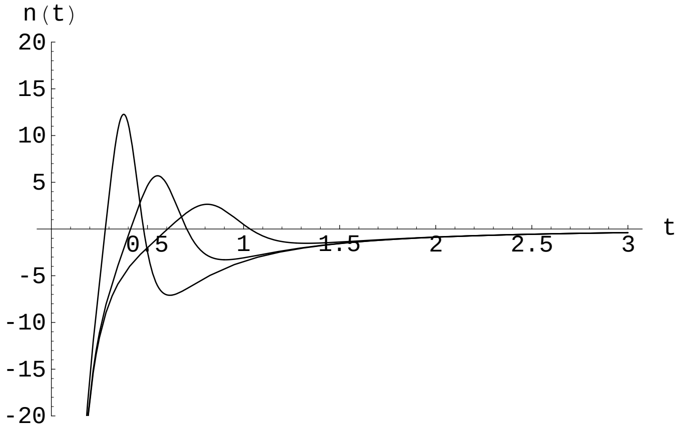

Figure 4: Plot of $n(t)$ for M-theory compactification with a gauge field given in (15) for $4\omega^6 = 0.01$, 0.1, and 1 and $\theta = \pi/4$. The left curve is for the value 0.01, the middle one for 0.1 and so on. Here $n(t)$ is positive in some finite time interval for different values of $4\omega^6$ chosen, showing the accelerating phase. But beyond these points there are deceleration.

in M-theory case or in Case I(a) above. We indeed find that to be true and so, we do not give those plots here. Here also the upper sign of (19) gives accelerated expansion but the lower sign gives deceleration.

(b) In this case we have $a = \delta = 0$ implying $\alpha = \beta = \pm 3/2$. Here also $\chi = 6$, but now we have $b \neq 0$ which implies $\theta \neq 0$. We therefore have a non-zero 5-form gauge field. The function F has now the form

$$F(t) = g(t)^{\pm\frac{3}{4}} \cos^2\theta + g(t)^{\mp\frac{3}{4}} \sin^2\theta \tag{20}$$

where $g(t)$ is as given in Case II(a) above. We have plotted $m(t)$ and $n(t)$ for three different values of θ namely, $\theta = \pi/4$, $\theta = \pi/6$ and $\theta = \pi/18$ with three different $4\omega^5 = 0.01, 0.1, 1$. Here also since this is pure gravity with a non-zero gauge field, we expect the functions to behave similarly as in Case I(b) above. We find this to be true and so, we do not give the plots here.

(c) In this case we have a non-zero dilaton and so, $a = -1/2$. But we choose $b = 0$ which implies $\theta = 0$ and the form field is vanishing. Here $\chi = 32/5$ and from (16) we find

$$\begin{aligned} \alpha &= \pm\sqrt{\frac{192 - 75\delta^2}{80}} - \frac{\delta}{4} \\ \beta &= \pm\sqrt{\frac{192 - 75\delta^2}{80}} + \frac{\delta}{4} \end{aligned} \tag{21}$$

The various functions defined in (13) and (14) now take the forms,

$$\begin{aligned} F(t) &= g(t)^{\frac{1}{2}\left(\pm\sqrt{\frac{192-75\delta^2}{80}} - \frac{\delta}{4}\right)} \\ A(t) &= g(t)^{\frac{1}{8}\left(\pm\sqrt{\frac{192-75\delta^2}{80}} - \frac{\delta}{4}\right) + \frac{3}{10}} t^3 \\ B(t) &= g(t)^{\frac{1}{4}\left(\mp\sqrt{\frac{192-75\delta^2}{80}} + \frac{\delta}{4}\right) + \frac{2}{5}} \end{aligned} \tag{22}$$

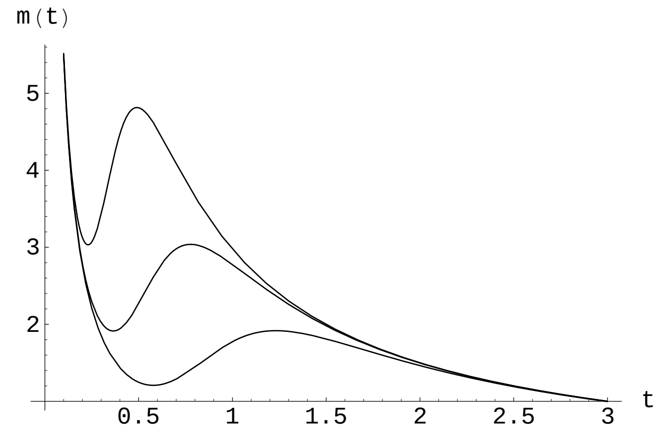

Figure 5: Plot of $m(t)$ given in (15) for string theory compactification with a dilaton but no gauge field ($\theta = 0$) for $4\omega^5 = 0.01$, 0.1, and 1 and $\delta = 0.1$. The top curve is for the value 0.01, the middle one for 0.1 and so on. All the curves meet at small and large values of t.

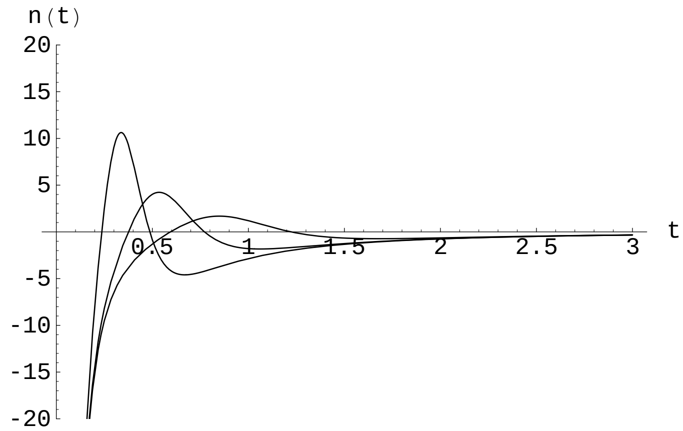

Figure 6: Plot of $n(t)$ given in (15) for string theory compactification with a dilaton but no gauge field ($\theta = 0$) for $4\omega^5 = 0.01$, 0.1, and 1 and $\delta = 0.1$. The left curve is for the value 0.01, the middle one for 0.1 and so on. All the curves are positive for certain interval of time indicating an accelerating cosmology. Beyond these points there are deceleration.

where $g(t)$ is as given before in Case II(a) and $|\delta| \leq \sqrt{192/75}$, but otherwise is an arbitrary parameter. We have plotted $m(t)$ and $n(t)$ as given in (15) for various values of δ and $4\omega^5$ in figures 5,6. We found like the parameter $4\omega^5$ that for not all values of δ we get an accelerating cosmology. We chose a specific value of $\delta = 0.1$ and plotted the functions for three different values of $4\omega^5$. For all these cases we get accelerating cosmologies in some time interval. Here we have used only the upper sign of the various functions given in (22). The lower sign gives deceleration. Since in this case we have a non-zero dilaton, we expect to have a different behavior of the functions. But surprisingly, the new parameter δ has little effect on the cosmology. The differences in behavior of the functions are indeed very small as can be seen by comparing figures 5,6 with those of the M-theory compactifications given in figures 1 – 4.

(d) In this case none of the parameters are zero. So, we have a non-zero dilaton as

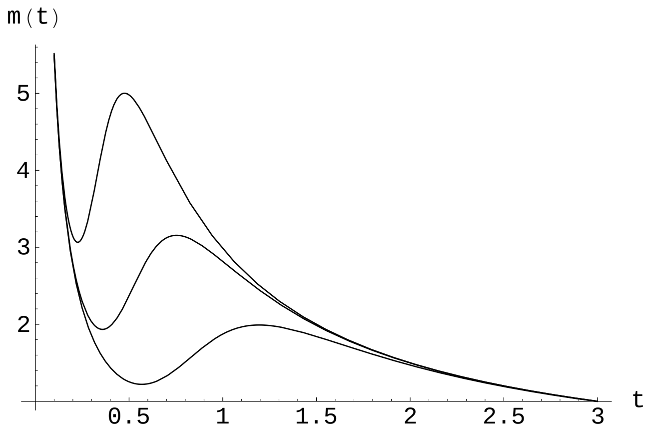

Figure 7: Plot of $m(t)$ given in (15) for string theory compactification with a dilaton and a gauge field for $4\omega^5 = 0.01$, 0.1, 1, $\delta = 0.1$ and $\theta = \pi/6$. The top curve is for the value 0.01, the middle one for 0.1 and so on. All the curves meet at small and large values of t.

well as a non-zero 5-form gauge field. Again for this case $\chi = 32/5$ and $a = -1/2$. α and β are the same as in Case II(c) above. The various functions are given as,

$$
\begin{aligned}
F(t) &= g(t)^{\frac{1}{2}\left(\pm\sqrt{\frac{192-75\delta^2}{80}}-\frac{\delta}{4}\right)}\cos^2\theta + g(t)^{\frac{1}{2}\left(\mp\sqrt{\frac{192-75\delta^2}{80}}-\frac{\delta}{4}\right)}\sin^2\theta \\
A(t) &= \left(g(t)^{\frac{1}{2}\left(\pm\sqrt{\frac{192-75\delta^2}{80}}-\frac{\delta}{4}\right)}\cos^2\theta + g(t)^{\frac{1}{2}\left(\mp\sqrt{\frac{192-75\delta^2}{80}}-\frac{\delta}{4}\right)}\sin^2\theta\right)^{\frac{1}{4}} g(t)^{\frac{3}{10}}\, t^3 \\
B(t) &= \left(g(t)^{\frac{1}{2}\left(\pm\sqrt{\frac{192-75\delta^2}{80}}-\frac{\delta}{4}\right)}\cos^2\theta + g(t)^{\frac{1}{2}\left(\mp\sqrt{\frac{192-75\delta^2}{80}}-\frac{\delta}{4}\right)}\sin^2\theta\right)^{-\frac{1}{2}} g(t)^{\frac{2}{5}} \qquad (23)
\end{aligned}
$$

We have plotted the functions $m(t)$ and $n(t)$ given in (15) for various values of θ, δ and $4\omega^5$. As before for not all values of the parameters we get accelerating cosmologies. We here give the plots for $\theta = \pi/6$, $\delta = 0.1$ and with three different values of $4\omega^5 = 0.01$, 0.1 and 1 in figures 7,8. With more parameters we expect the functions $m(t)$ and $n(t)$ to behave differently from the other cases, but again we found that the additional parameters have very little effect on the cosmology and we get essentially the same behavior as in the other cases. We have plotted the functions in various cases with the same scale for better comparison.

To summarize we have studied the conditions for accelerating cosmologies (15) for various M/string theory compactifications on hyperbolic space with time varying volume. In all the cases we have seen that both the conditions in (15) can be simultaneously satisfied in a certain time interval for some specific values of the various parameters. This indicates that under such compactifications the resulting four dimensional isotropic, homogeneous, FLRW universe in the Einstein frame can support accelerating cosmologies for certain period of time. This acceleration is transient with an e-folding of the order of one and does not lead to an interesting cosmological scenario. In order to obtain the accelerating cosmology we have made use of the time dependent supergravity solution in the form of S-

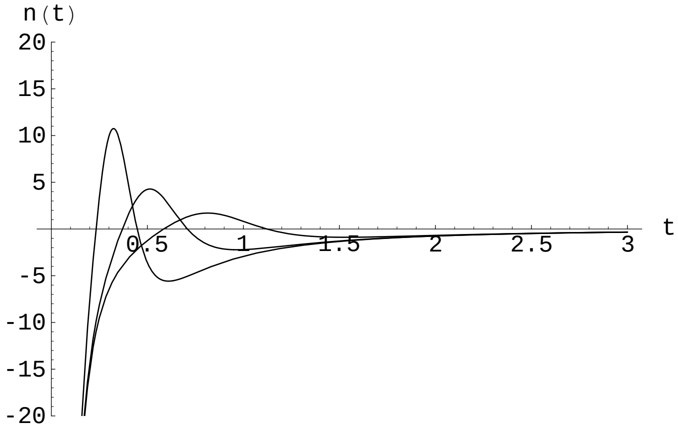

Figure 8: Plot of $n(t)$ given in (15) for string theory compactification with a dilaton and a gauge field for $4\omega^5 = 0.01$, 0.1, 1, $\delta = 0.1$ and $\theta = \pi/6$. The left curve is for the value 0.01, the middle one for 0.1 and so on. All the curves are positive for certain interval of time indicating an accelerating cosmology. Beyond these points there are deceleration.

branes. A similar phenomenon was known for a class of S-branes which are asymptotically non-flat [3, 4]. However, here we have used another known class of S-branes which are asymptotically flat [5, 6]. Here we remark that although the details of the four dimensional cosmology we obtain from these two classes are quite different, which are also seen from the figures given in this paper and those obtained in the earlier literature [15 − 17], the robust features are quite similar. We here make a few comments regarding the various cases we have studied with the asymptotically flat solution. First of all, we have given eight figures, four for M-theory and four for string theory compactifications. All the cases are different with different values of the parameters, yet the figures look quite the same. From all the plots of $m(t)$ we see that they remain positive for all values of t indicating that they give expanding universe. Also they have two extrema, one maxima and one minima. The positions of these extrema and the variations of the curves change significantly with the change of the parameter $4\omega^6$ (for M-theory) or $4\omega^5$ (for string theory). But beyond certain time interval the function $m(t)$ changes very little with $4\omega^6$ or $4\omega^5$. This is the reason the plots $n(t)$ have two zeroes. So, the cosmologies start with deceleration (corresponding to negative values of $n(t)$) and also end in deceleration, but since they have two zeroes we have accelerating phase in between. However, since the positions of extrema changes with $4\omega^6$ or $4\omega^5$, the period of acceleration also changes with the parameter. But we must remark that the peroid of acceleration does not change much with the introduction of other parameters (like δ and/or θ). It is easy to see that the relation between the actual four-dimensional time η and t given in eq. (11) can be integrated for large ($t \to \infty$) and small ($t \to 0$) t. For $t \to \infty$ we find $t \sim (\eta - \eta_0)^{2/(n+2)}$ and so, from (13) we get the late time behavior of the scale factor as $S(\eta) \sim (\eta - \eta_0)^{n/(n+2)}$. So, the behavior depends on the theory from which the cosmology is obtained, but the detailed structure is not important. However, keeping the upper signs of α and β given in (16), it is easy to check that at early time $F(t) \sim t^{-(n-1)\alpha/2}$ and $t \sim \eta^{-4\chi/(6\alpha(n+2)-3n\chi)}$. Similarly for the lower signs of

α and β, we can check that at early time, $F(t) \sim t^{(n-1)\beta/2}$ and $t \sim \eta^{4\chi/(6\beta(n+2)+3n\chi)}$. For both cases the behavior of the scale factor at early time can be seen from (13) to have the form $S(\eta) \sim \eta^{1/3}$. Now we find that the behavior is quite universal independent of n. These early and late time behaviors of the scale factor match exactly with the pure gravity compactification (although their solution is different from the one used here) observed by Townsend and Wohlfarth [15].

The universal feature of the scale factor at early time can be attributed to the fact that the metric takes the generalized Kasner form when $t \to 0$ and the resulting space-time is four dimensional. We would like to remark that the solution we have used in this paper is asymptotically flat with a 'horizon' at $t = 0$, just like the static, BPS p-brane (with horizon at $r = 0$) and since the near horizon geometry of a static D3-brane has the anti de Sitter structure, similarly, one might expect to get a de Sitter structure for this time dependent S2-brane solution in the near 'horizon' limit. But, it can be easily checked for the solution (8) that, we get de Sitter structure only if δ is a complex number or in other words a real de Sitter solution does not follow from the near 'horizon' limit of S2-brane. However, we find that $t \to 0$ limit of the metric and the dilaton in (8) have the forms[10],

$$\begin{aligned} ds^2 &= -t^{-\frac{6\alpha}{\chi}+n-2}dt^2 + t^{\frac{2\alpha}{\chi}(n-1)}\sum_{i=1}^{3}(dx^i)^2 + t^{-\frac{6\alpha}{\chi}+1}dH_n^2 \\ e^{2\phi} &= t^{\frac{2a\alpha(n+2)}{\chi}-\delta(n-1)} \end{aligned} \tag{24}$$

Since in the above t is nearly zero, one can rewrite the metric by replacing t by λt with $\lambda \to 0$ and $t =$ finite, in the form

$$ds^2 = \lambda^{-\frac{6\alpha}{\chi}+n}\left[-t^{-\frac{6\alpha}{\chi}+n-2}dt^2 + \lambda^{\frac{2\alpha}{\chi}(n+2)-n}t^{\frac{2\alpha}{\chi}(n-1)}\sum_{i=1}^{3}(dx^i)^2 + \lambda^{1-n}t^{-\frac{6\alpha}{\chi}+1}dR_n^2\right] \tag{25}$$

Note that we have replaced dH_n^2 in (24) by the line element of a flat n-dimensional Euclidean space dR_n^2. This is possible because for $n > 1$, the coefficient of dH_n^2 in (24) becomes infinite and so we can replace it by the flat space. Going back to the original coordinate t we therefore write the solution (24) by replacing dH_n^2 by dR_n^2. Now defining a new coordinate by $d\bar{t} = t^{-(3\alpha/\chi)+(n/2)-1}dt$ we can write the metric and the dilaton in (24) as,

$$\begin{aligned} ds^2 &= -d\bar{t}^2 + \bar{t}^{-\frac{4\alpha(n-1)}{6\alpha-\chi n}}\sum_{i=1}^{3}(dx^i)^2 + \bar{t}^{\frac{2(6\alpha-\chi)}{6\alpha-\chi n}}dR_n^2 \\ e^{2\phi} &= \bar{t}^{\frac{-4a\alpha(n+2)+2\delta\chi(n-1)}{6\alpha-\chi n}} \end{aligned} \tag{26}$$

Further, defining the various exponents of $\bar{t}$ appearing in the metric and the dilaton in (26)

$$p = -\frac{2\alpha(n-1)}{6\alpha-\chi n}, \qquad q = \frac{6\alpha-\chi}{6\alpha-\chi n}, \qquad \gamma = \frac{-2a\alpha(n+2)+\delta\chi(n-1)}{6\alpha-\chi n} \tag{27}$$

[10]Here we keep only the upper signs of α and β given in (16) and mention about the lower signs later. Also since for very low values of t, $4\omega^{n-1}$ has very little effect on cosmology we have put $4\omega^{n-1} = 1$ without any loss of generality.

we find that they satisfy

$$3p + nq = 1, \qquad 1 - 3p^2 - nq^2 = \frac{1}{2}\gamma^2 \tag{28}$$

In order to satisfy the second equation of (28) we have made use of the relation between the parameters α and δ given in (2). These are precisely the conditions satisfied by the generalized Kasner metric [26]. Note that in deriving (24) and (26) we have used the upper signs of α, β, however, for the lower signs, they have very similar forms with α replaced by $-\beta$. The various exponents of $\bar{t}$ in that case have the forms

$$p = -\frac{2\beta(n-1)}{6\beta + \chi n}, \qquad q = \frac{6\beta + \chi}{6\beta + \chi n}, \qquad \gamma = -\frac{2a\beta(n+2) + \delta\chi(n-1)}{6\beta + \chi n} \tag{29}$$

It is easy to check that they again satisfy eq. (28) if we use eq. (2). We thus conclude that the near 'horizon' limit of the solution (8) is the generalized Kasner metric. Note that when there is no dilaton $\gamma = 0$, $a = 0$ and $\delta = 0$, then p and q reduce precisely to the standard Kasner form given in [15]. Because of this the scale factor of the resulting four dimensional FLRW cosmology has a universal behavior $S(\eta) \sim \eta^{1/3}$.

We have thus seen that the four dimensional accelerating cosmologies quite generically follow from the known S-brane solutions of M/string theory. This was known for the asymptotically non-flat solutions and we have shown this to be true for the asymptotically flat solutions as well. As in the former case, we have seen that the cosmology does not change much with the introduction of various parameters. Although we get accelerating cosmologies in various cases, the acceleration is transient. Since S-branes are unstable systems without any supersymmetry, it might be interesting to see whether by coupling the four dimensional action with the tachyon effective action and/or the inclusion of the higher order curvature terms can give a better understanding of the initial cosmological singularity and a longer period of acceleration leading to a de Sitter solution.

Acknowledgments

H.S. is grateful to AS-ICTP, Trieste and its Associateship Programme for hospitality where part of this work has been carried out.

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